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EFFECT OF ALIASING ON SPURS AND PM NOISE IN FREQUENCY DIVIDERS*

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ABSTRACT

We show that the noise and spurious signals over the entire input frequency response of a binary digital divider contribute with equal weight to the output signal. Similar results are also obtained using non-binary digital dividers. We present a simple model that allows one to understand the mechanisms that produce these aliasing effects. Only filters at the input or intermediate stages of division can reduce these effects in digital dividers. We also show that direct-digital-synthesis where the output of a counter is used to drive a shift register and a digital-to-analog converter to produce a sinewave output is largely free from these effects.

INTRODUCTION

Digital dividers are widely used in a wide range of applications such as phase-locked-loop oscillators, where a high frequency oscillator is phase-locked to a low frequency reference using a divider. In this scheme the close-to-carrier phase modulation (PM) noise is the sum of the PM noise of the reference plus N^2 times the output PM noise of the divider, where N is the division ratio. For large division ratios the PM in the dividers often determines the PM noise of the output signal.

In this paper we develop a simple analytical model to understand the mechanisms that cause the increase in PM noise in high-division-ratio digital dividers. Once the mechanisms are understood it is then possible to consider possible remedies. The same mechanisms that increase the PM noise of a digital divider also cause the spurs in the input signal to be folded into the output spectrum. Our model provides an easy method to determine the position of these folded (or aliased) spurs.

The intrinsic output PM noise of a noiseless divider is given by

$$L(\upsilon_o, f) = L(\upsilon_{in}, f) - 20 \text{ Log } N, \qquad (1)$$

where $L(\upsilon_o, f)$ is the output single sideband PM noise at frequency υ_o, $L(\upsilon_{in}, f)$ is the input PM noise at frequency υ_{in}. We show that in digital frequency dividers the PM noise and spurious signals over the entire input frequency response of the divider contribute with nearly equal weight to the output signal at υ_o. This is true even when their separation from the original carrier is large compared to the final output frequency, and an output filter is used to eliminate the harmonics created by the digital divider. This is in contrast to dividers that use a phase accumulator with a lookup table to synthesize a sinewave output signal, where these effects are minimal.

The PM noise degradation is due to aliasing in the intermediate stages of the divider. The spurious signals and added noise are always found to be symmetrically placed about the output signal and its harmonics [1]. We show a simple relationship to determine the amplitude and location of these spurious signals in the output frequency based on their initial amplitude and separation from the input signal. If the noise bandwidth (BW) of the input signal is more than 4 times wider than the final output signal υ_o, the output will contain substantial excess wideband noise approximately given by

$$\text{Excess PM} = 10 \log [(BW/2\upsilon_o) + 1]. \quad (2)$$

For high division ratios the excess noise can easily be 10 to 20 dB higher than the intrinsic noise given by Eq. (1). The aliasing effect depends only on the input noise BW and the output frequency and is independent of the actual value of N. Aliasing of the broadband noise generally has a much smaller effect on the close-to-carrier noise because it is typically many orders of magnitude higher than the wideband noise. Eagan has treated the problem from sampling theory and obtained similar results for broadband noise, but did not treat spurs [2].

We demonstrate these aliasing effects by measuring the PM noise of a divide by 32 circuit and a divide by 11 circuit, as a function of the bandwidth of the input noise. Both had an output frequency of 10 MHz. When the input noise bandwidth is 15 MHz or less we obtain PM noise at 10 MHz given by (1), i.e., there is no significant aliasing. When the input noise bandwidth BW about the input signal is 70 MHz, the 10 MHz wideband PM noise is 9 dB higher than that given by (1). When the input noise BW is 500 MHz, the 10 MHz wideband PM noise is 14 dB higher than that given by (1). Both of these results agree very well with (2). If the frequency response of the divider were 10 GHz and the noise were also that wide, then the aliasing effect would add 27 dB of excess wideband PM noise to the 10 MHz output signal.

These results show that aliasing effects in dividers can have a profound effect on the output broadband PM noise and spurious content.

BINARY DIGITAL DIVIDERS

To simplify the discussion we first consider a binary divider. The output of a typical binary divider is a square wave with fast rise and fall times. This results in an output spectrum at the fundamental, a large number of odd harmonics, and little power in the even harmonics. Specifically, the idealized output spectrum is given by

$$V(t) = Vo[\sin(2\pi\upsilon_o t) + 1/3 \sin(2\pi 3\upsilon_o t) + 1/5 \sin(2\pi 5\upsilon_o t) \ldots \quad (3)$$

Note that the power in the signal, which is proportional to Vo^2, decreases as $1/n^2$, where n is the harmonic number. Since the PM noise of the harmonics must scale as n^2, the amplitudes of the noise sidebands and spurious signals about the harmonics of the output frequency, are the same amplitude as those of the sidebands about the fundamental.

A spurious signal about the input signal of separation f and fractional amplitude $BETA^2$ appears about the output signal at the same separation f and at a fractional amplitude $[BETA^2/(2N)]^2$. This occurs because a single-sideband spurious signal at the input is a special combination of equal AM and PM at separation f above and below the carrier [2]. Therefore the true power in the single-sideband PM noise at the input is $[BETA/2]^2$

The net effect is that all the noise and spurious signals at separations f_i, f_{i+1}, f_{i+2},...appear about the output signal and all its odd harmonics at +/- f_i, f_{i+1}, f_{i+2},...about the input signal. Moreover the noise and spurs at f_i appear at an amplitude of $[BETA_i/(2N)]^2$ about each odd harmonic of the output frequency. As the spur or noise signal separation from the carrier increases, it wraps around 0 and again increases because we cannot distinguish –f from f.

To illustrate this further, consider an input signal frequency υ_{in} of 320 MHz and an output frequency of a 1/32 divider of 10 MHz. A spurious signal at 320 +1 MHz will appear at the output frequencies of 9 MHz and 11 MHz, 29 MHz and 31 MHz due to the 3rd harmonic, 49 MHz and 51 MHz due to the 5th harmonic and so forth. A spurious signal at 320 +19 MHz will appear in the output at frequencies of 9 MHz (actually –9 MHz) and 29 MHz, 11 MHz and 49 MHz due to the 3rd harmonic, 31 MHz and 69 MHz due to the 5th harmonic.... A spurious signal at 320 +41 MHz will appear in the output at frequencies of -31 MHz and 51 MHz, -11 MHz and 71 MHz due to the 3rd harmonic, 9 MHz and 91 MHz due to the 5th harmonic..... The folding of the spurs and noise about the input signal is illustrated in Figure 1.
This process continues for noise and spurious signals over the entire input bandwidth on the divider. We see that only the **lower** sidebands contribute to

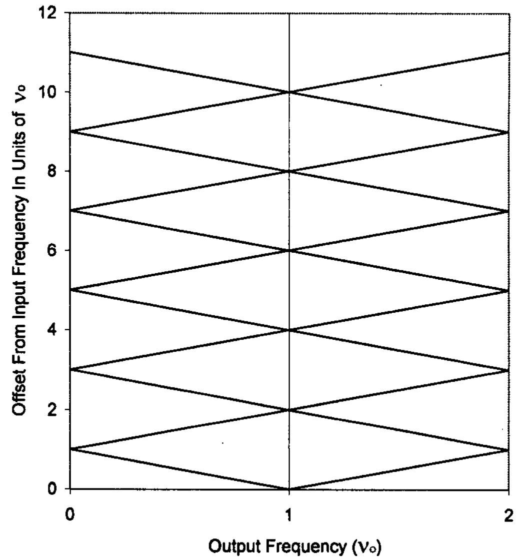

Figure 1. Folding of noise and spurs as a function of offset from the input carrier signal.

excess noise and spurs. The lower sideband of the fundamental contributes only once to the excess noise, while the lower sidebands of the harmonics of the output signal contribute twice to the output signal, once when the difference between the harmonic and the spurious frequency is equal to + υ_o and once when the difference is equal to - υ_o. For broadband noise, the contributions from the harmonics will be approximately equal over the input bandwidth of the divider. The use of a post filter at the output of the divider would remove the harmonics but would do nothing to remove the excess noise and spurious signals that have been aliased about υ_o. The summation of these noise contributions then leads to the result given in Eq. (2)

Noise and spurious signals about the harmonics of υ_{in} are alaised to υ_{in} and contribute to the output signal in the same manner described above, limited only by the input bandwidth of the divider. If the frequency response of the divider were 10 GHz and the noise were also that wide, then the aliasing effect would add 27 dB of excess wideband PM noise to the 10 MHz output signal.

NON-BINARY DIGITAL DIVIDERS

The results of the previous discussion on binary dividers can be generalized to other division ratios. In non-binary divider stages there are even as well as odd harmonics of the output frequency. The lower sidebands of noise and spurious signals about the even harmonics provide additional contributions to the output signal. The generalized output of a non-binary divider can be expressed as

$$V(t) = V_0 \Sigma\, k(n) \sin(2\, \pi n \upsilon_o t), \qquad (4)$$

where k(n) are the coefficients of the nth harmonics of υ_o. This leads to a generalized expression for the output SSB phase noise of

$$L(v_o,f) = \Sigma\, \{n\, k(n)/N\}2\, \{L(\upsilon_{in},\{n-1\}\upsilon_o + f) + \{L(\upsilon_{in},\{n+1\}\upsilon_o + f). \qquad (5)$$

DIRECT DIGITAL SYNTHESIS

Some synthesis schemes use the output of a counter to increment a phase register. The output of the phase register then uses a digital-to-analog converter to produce the output sine wave signal. A low-pass filter removes most of the high frequency noise and harmonics. In this scheme we do not expect to see any aliasing of noise about υ_{in}, only noise about the harmonics of υ_{in}, because there no intermediate signals to participate in the lower sideband aliasing.[3] This is verified in the experimental test described below.

EXPERIMENTAL TESTS OF ALAISING IN BINARY AND DDS DIVIDERS

Figure 2 shows a simple scheme to measure the aliasing of spurs in binary dividers. The input signal at 320 MHz is divided by 32 to 10 MHz. An auxiliary signal generator that can be adjusted over a wide frequency range is used to add a SSB spur to the signal. The input spectrum shown in Fig. 3 with a SSB spur at + 1.58 MHz creates the output spectrum shown in Fig. 4. The SSB input spur produces both an upper and a lower PM sideband in the output. The amplitude of the output spurs is within 0.2 dB of that given by the discussion under the binary divider section.

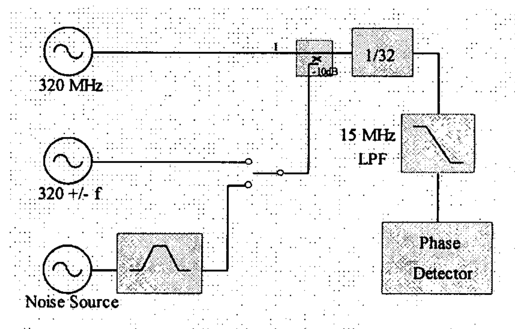

Figure 2. Block diagram of system to measure aliasing of spurious signals (switch up) and noise (switch down).

Figure 5 shows another input spectrum with a SSB spur 21.58 MHz above the input signal. The output spectrum is the same as that for the input spectrum of Fig.2. The 21.58 MHz spur has been alaised to 1.58 MHz about the output 10 MHz signal.

Tests of wideband noise alaising were carried out by replacing the auxiliary signal generator of Fig. 2 by an adjustable-bandwidth noise source that was centered at 320 MHz. Figure 6 shows the input 320 MHz power spectrum for a noise BW of approximately 15 MHz. The output PM noise was then measured as a function of the BW of the noise. Table 1 shows the results. The estimated values in column 3 were derived from Eq. 2. The agreement with Eq. 2 is well within the estimated uncertainty of ± 0.5 dB for the measurements.

Similar tests were run on DDS-based dividers. In this case we found no evidence of alaising of either spurs or wideband noise as long as the input included a low pass filter to excluded harmonics of the input frequency.

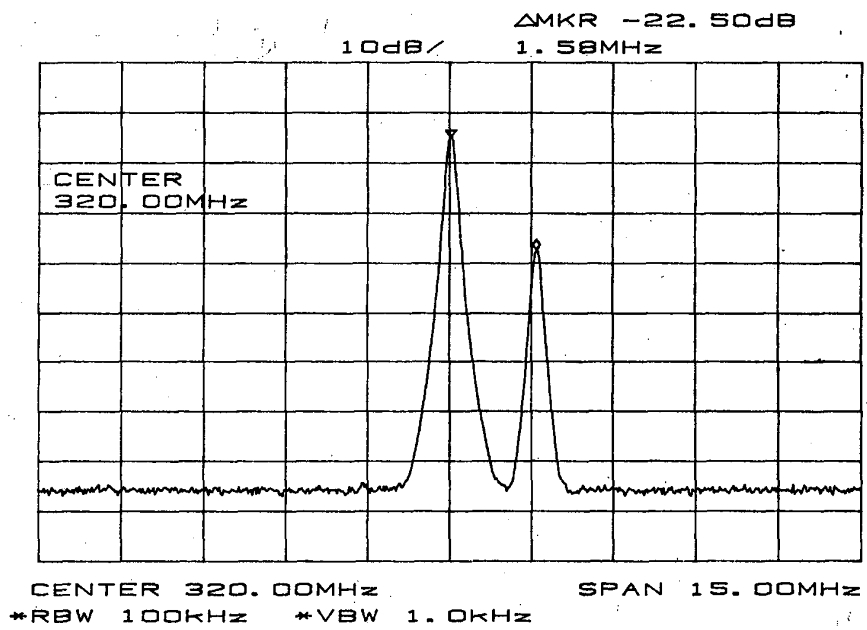

Figure 3. Input signal at 320 MHz showing a spur at an offset of + 1.58 MHz.

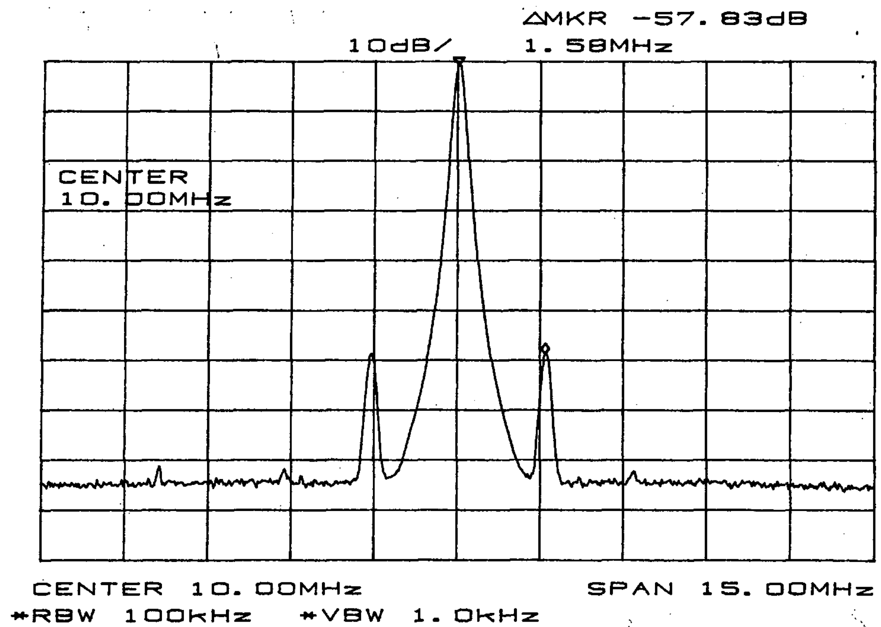

Figure 4. Output signal at 10 MHz generated by the input spectrum of Fig. 3. The spurs are at offsets of ± 1.58

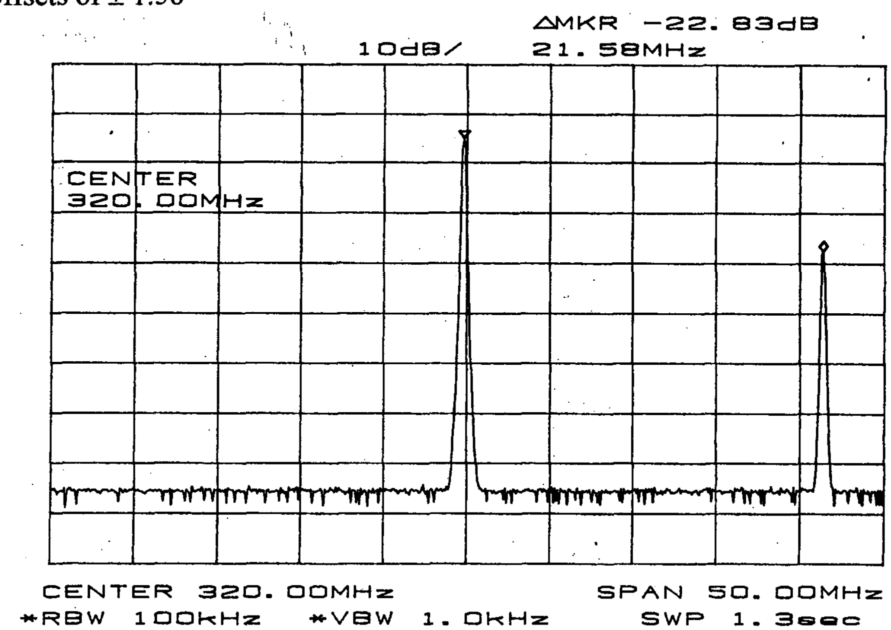

Figure 5. Input signal at 320 MHz showing a spur at an offset of + 21.58 MHz.

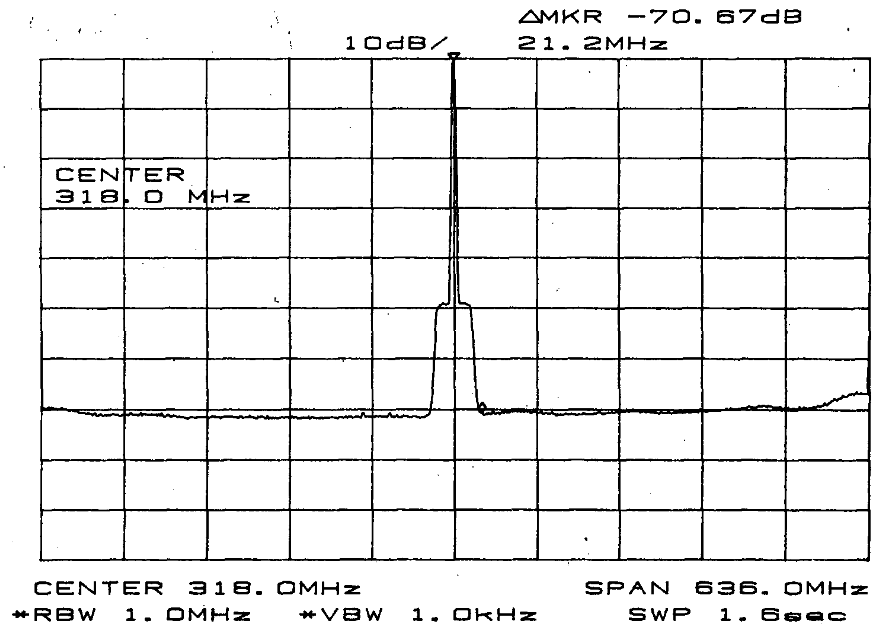

Figure 6. Input signal at 320 MHz showing a flat noise spectrum of width approximately 15 MHz

DISCUSSION

PM noise at the output of a digital divider can be enhanced considerable due to aliasing. The effect is most serious for large division ratios. Low pass filtering of the output signal does not help because the lower sidebands of the harmonics of the output frequency are already mixed with the signal of interest. The only way to reduce the problem is to use band-pass filters at the input or at one or more intermediate frequencies. For example a 10 % bandwidth filter on the input signal would permit division by 64 before aliasing became serious. Adding another 10 % filter at the output of the first divide by 64 stage would permit an additional division of up to 64 (total division of 4096) before aliasing became a concern.

Another alternative for reducing the problem is to use a DDS approach. If the input frequency is too high to implement this approach, then a modest input filter bandwidth coupled with a DDS division for the last stages of division could reduce the problem considerably.

REFERENCES

[1] F. L. Walls, "Correlation between upper and lower sidebands" IEEE Trans. UFFC, Vol. 47, pp 407-410, 2000.

[2] William F. Eagan, "Modeling phase noise in frequency dividers," IEEE Trans. On UFFC, 37, pp. 30 7-315, 1990.

[3] Franklin G. Ascarrunz,"Frequency and phase offset signal generator and method" patent pending S/N 09/083879, May 98.

Table 1. Divider output PM noise 100 kHz from the carrier

Noise BW	Measured L(100 kHz)	Estimated L(100 kHz) from Eq. (2)
MHz	dBc/Hz	dBc/Hz
3.5	-58.0	-58.2
5	-58.4	-58.2
10	-58.0	-58.2
15	-58.2	-58.2
70	-49.0	-49.5
100	-46.9	-47.2
150	-45.6	-45.9
250	-44.2	-44.0